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ON TERNARY QUADRATIC DIOPHANTINE EQUATION $w^2 - 2z^2 + 2wx + 20zx = 51x^2$

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ABSTRACT:

The cone represented by the ternary quadratic Diophantine equation $w^2 - 2z^2 + 2wx + 20zx = 51x^2$ is analyzed for its patterns of non-zero distinct integral solutions.

KEYWORDS: Ternary quadratic, Homogeneous quadratic, Integral solutions

INTRODUCTION:

The Diophantine equation offers an unlimited field for research due to their variety [1-3]. In particular, one may refer [4-15] for quadratic equations with three unknowns. This communication concerns with yet another interesting equation $w^2 - 2z^2 + 2wx + 20zx = 51x^2$ representing homogeneous quadratic with three unknowns for determining its infinitely many non-zero integral points.

METHOD OF ANALYSIS:

The given ternary quadratic diophantine equation is

$$w^2 - 2z^2 + 2wx + 20zx = 51x^2 \tag{1}$$

On completing the squares,(1) is written as

$$P^2 - 2Q^2 = 2x^2 \tag{2}$$

where

$$P = w + x, Q = z - 5x \tag{3}$$

Let us see the different patterns of solving the above equation (3) and thus,

obtain the different choices of x, w and z satisfying (1).

Choice I:

Let us assume

$$x = a^2 - 2b^2 \quad (4)$$

We can write 2 as

$$2 = (2+\sqrt{2})(2-\sqrt{2}) \quad (5)$$

Using (4) and (5) in (2) and employing the method of factorization ,consider

$$P + \sqrt{2}Q = (2+\sqrt{2})(a+\sqrt{2}b)^2 \quad (6)$$

Equating the rational and irrational parts,the values of P and Q are obtained.

In view of (3),one has

$$w = a^2 + 6b^2 + 4ab, z = 6a^2 - 8b^2 + 4ab \quad (7)$$

Thus,(4) and (7) represent the integer solutions to (1).

Note 1:

In addition to (5),one may have

$$2 = (10+7\sqrt{2})(10-7\sqrt{2}),$$
$$2 = \frac{(10+\sqrt{2})(10-\sqrt{2})}{49}$$

Following the above procedure ,we have two more integer solutions to (1).

Choice II:

Write (2) as

$$P^2 - 2Q^2 = 2x^2 * 1 \quad (8)$$

Take 1 on the R.H.S. of (8) as

$$1 = (3+2\sqrt{2})(3-2\sqrt{2}) \quad (9)$$

Using (4),(5) & (9) in (8) and employing the method of factorization ,consider

$$P+\sqrt{2}Q=(10+7\sqrt{2})(a+\sqrt{2}b)^2 \tag{10}$$

Equating the rational and irrational parts,the values of P and Q are obtained.

In view of (3), one has

$$w=9a^2+22b^2+28ab,\ z=12a^2+4b^2+20ab \tag{11}$$

Thus,(4) and (11) represent the integer solutions to (1).

Note 2:

The integer 1 on the R.H.S. of (8) is also taken as

$$1=\frac{(2r^2+s^2+\sqrt{2}\,2rs)(2r^2+s^2-\sqrt{2}\,2rs)}{(2r^2-s^2)^2},$$

$$1=\frac{(11+6\sqrt{2})(11-6\sqrt{2})}{49},$$

$$1=(17+12\sqrt{2})(17-12\sqrt{2})$$

Following the above procedure ,three different solutions to (1) are obtained.

Choice III:

In (2) , the substitution

$$Q=u+v,\ x=u-v \tag{12}$$

leads to

$$P^2=4u^2+4v^2$$

which is satisfied by

$$u=4rs,\ v=2(r^2-s^2),\ P=4(r^2+s^2),\ r\geq s\geq 0$$

In view of (12) and (3) ,the corresponding integer solutions to (1) are given by

$$x = 4rs - 2r^2 + 2s^2,$$
$$w = -4rs + 6r^2 + 2s^2,$$
$$z = 24rs - 8r^2 + 8s^2$$

Choice IV:

In (2) , the substitution

$$Q = u + v, x = u - v, P = 2R \quad (13)$$

leads to

$$R^2 = u^2 + v^2$$

which is satisfied by

$$u = 2rs, v = (r^2 - s^2), P = (r^2 + s^2), r \geq s \geq 0$$

In view of (13) and (3) ,the corresponding integer solutions to (1) are given by

$$x = 2rs - r^2 + s^2,$$
$$w = -2rs + 3r^2 + s^2,$$
$$z = 12rs - 4r^2 + 4s^2$$

CONCLUSION

In this paper, we have made an attempt to obtain all integer solutions to (1). To conclude, one may search for integer solutions to other choices of ternary quadratic diophantine equations.

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